A quantum oscillator model for entropic gravity

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Abstract

In Verlinde's theory of entropic gravity, the internal degrees of freedom are marked by "bits", but the microscopic origin of "bits" is unknown hitherto. To this direction, we adopt the quantum harmonic oscillators to naively model the internal degrees of freedom. In this model, the equipartition relation is modified at the low temperature. The Newton's law and Poisson's equation of gravity are reproduced at small distances, but they receive appreciable corrections at large distances. Geometrization of gravity for this model is explored, resulting in modifications to Einstein equations.

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I. MOTIVATION AND MODEL

It remains a glamorous challenge today to uncover the quantum nature of gravity. One potential way to circumvent the challenge is acquiring gravitational equations from thermodynamics of spacetime [1]. Along this way, Verlinde proposed to interpret gravity as an entropic force [2]. The idea turns out to be powerful. From it one can derive both Newton's laws and Einstein's equations [2]. The derivation of gravitational equations in [2] is based on three ingredients: the Unruh law [3] identifying temperature with the local acceleration; the holographic principle ensuring that the number of bits is proportional to the area of holographic screen; and an equipartition rule [4] relating energy or mass to the temperature and the number of bits.

In Verlinde's scenario, "bits" are used to characterize the microscopic degrees of freedom. However, the microscopic picture of"bits" is unclear. Moreover, given a concrete picture, the aforementioned three ingredients may be modified. For example, it is well known that the equipartition theorem works well in classical statistical mechanics, but it becomes inaccurate when quantum effects turn up, such as at low temperatures. If this happens in the scenario of entropic gravity, the quantum effects will enter into gravity at low energies and large distances.

Harmonic oscillators, being one of the simple models in quantum mechanics, have been utilized by Einstein to model the internal degrees of freedom when explaining the specific heat of solids. In the Einstein model, each atom in a solid is an independent 3-dimensional quantum harmonic oscillator. This leads us to propose a naive but concrete microscopic picture for entropic gravity. In the picture, microscopic degrees of freedom of gravity will be modeled by 1-dimensional quantum harmonic oscillators with the same frequency, and the number of bits is proportional to the degrees of freedom up to a redefinition of Newton's constant. Interestingly, the energy of quantum oscillator is quantized and has a nonvanishing value at zero temperature, seeding quantum effects in our model of entropic gravity. The effects will be evident in modified gravitational equations.

We are aware that this model is a premitive toy model. Related to our attempt, a deep problem is the quantum origin of entropic gravity. This is an attractive but hard problem. In the literature [5–10], it has been explored in various directions with some interesting results.

In this paper we will study implications of the above quantum oscillator model to entropic gravity. The modified Newton's law of gravity is shown in Sec. II. After generalizing Poisson's equation in Sec. III, we will work out the relativistic version of gravitational equations in Sec. IV A and study its Newtonian limit in Sec. IV B. Sec. V is a brief summary and discussion. In Appendix A, we work out the expression of mass from the modified equipartition relation in detail. Most of our results hold not only for the quantum harmonic oscillator model, but also for more general models of modified entropic gravity.

II. MODIFIED NEWTONIAN GRAVITY

Along the line of [2], an Unruh temperature T is required to cause an acceleration a through the relation

$$k_B T = \frac{\hbar a}{2\pi c}. \tag{1}$$

As an assumption in [2], the internal degrees of freedom in the equipartition rule are

equal to the number of "bits" which is proportional to the area of holographic screen. In our case, we have N harmonic oscillators, but the harmonic oscillator has an infinite number of levels, so the number of bits cannot be equated to N. However, remember that the number of bits, like entropy, is proportional to N, then we can relegate the proportional coefficient into the redefinition or renormalization of Newton's constant, and write down

$$N = \frac{Ac^3}{2G\hbar}, \tag{2}$$

where G is the renormalized Newton's constant.

From statistical physics, the total energy of N harmonic oscillators with the same frequency ω is

$$E = \frac{1}{2}N\hbar\omega + \frac{N\hbar\omega}{e^{\hbar\omega/k_BT} - 1}. \tag{3}$$

This relation reduces to $E = Nk_BT$ in the high temperature limit $\hbar\omega \ll k_BT$. This is easy to understand because every oscillator has two quadratic terms in the Hamiltonian, a kinetic energy term and a potential energy term. For convenience, let us define a constant $a_E = 2\pi\omega c$ and refer to this constant as Einstein acceleration. Utilizing (1), (2) and the relation $E = Mc^2$, we solve equation (3) analytically, obtaining the acceleration

$$\begin{aligned} a &= \frac{a_E}{\ln\left(\frac{1}{\frac{2GM}{Ac\omega} - \frac{1}{2}} + 1\right)} \\ &= \frac{a_E}{\ln\left(\frac{1}{\frac{GM}{r^2 a_E} - \frac{1}{2}} + 1\right)}. \end{aligned} \tag{4}$$

In figure 1, this equality is illustrated and compared with Newton's law of gravity. Equality (4) traces Newton's law at short distances but deviates from it at large distances. Especially, the gravity is not well-defined if $r^2 > 2GM/a_E$ according to (4). This appears to be disastrous at first sight. However, the disaster never happens. The zero point energy, given by the first term of (3), always induces a mass $r^2 a_E/(2G)$ inside a holographic screen of radius r. Therefore, it is guaranteed that $r^2 \leq 2GM/a_E$ if we take into account the zero-point energy.

It is interesting to expand (4) in the small radius limit $r^2 \ll GM/a_E$, yielding

$$a \simeq \frac{GM}{r^2} - \frac{r^2 a_E}{12GM} \tag{5}$$

next to the leading order. This indicates that gravity is weakened in our model, similar to the behavior of a positive cosmological constant. For comparison, in figure 1 we have also displayed a modified Newton's law [11, 12]

$$a = \frac{GM}{r^2} - \frac{\Lambda r}{3}. \tag{6}$$

with cosmological constant Λ.

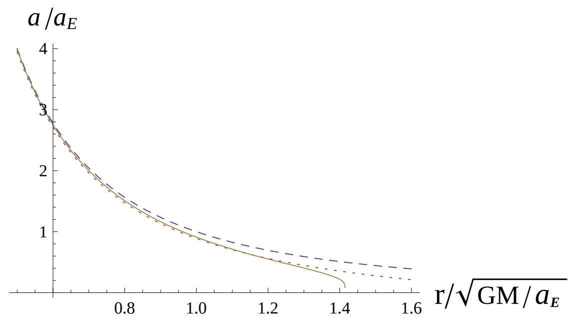

Figure 1: (color online). An illustration of equality (4) (solid brown curve) versus Newton's law(dashed blue curve without cosmological constant, and dotted purple curve with cosmological constant $\Lambda = a_E\sqrt{a_E}/\sqrt{9GM}$).

III. GENERALIZED POISSON'S EQUATION

Parallel to [2], by writing (2) in the differential form

$$dN = \frac{c^3}{2G\hbar}dA, \tag{7}$$

we can get an integral form of the equipartition relation (3) on the holographic screen $\mathcal{S}$, that is

$$\begin{aligned} E &= \int_{\mathcal{S}} \left(\frac{1}{2}\hbar\omega + \frac{\hbar\omega}{e^{\hbar\omega/k_BT}-1}\right) dN \\ &= \frac{c^3}{2G\hbar}\int_{\mathcal{S}} \left(\frac{1}{2}\hbar\omega + \frac{\hbar\omega}{e^{\hbar\omega/k_BT}-1}\right) dA. \end{aligned} \tag{8}$$

As a vector, the acceleration $\vec{a} = -\nabla\Phi$, where Φ is the Newtonian potential. Therefore, relation (1) can be put into the form

$$k_BT = \frac{\hbar}{2\pi c}|\nabla\Phi|. \tag{9}$$

Remember also that $E = Mc^2$ and $a_E = 2\pi\omega c$, so equation (8) will take the form

$$M = \frac{1}{4\pi G}\int_{\mathcal{S}} \left(\frac{1}{2} + \frac{1}{e^{a_E/|\nabla\Phi|}-1}\right)\frac{a_E}{|\nabla\Phi|}\nabla\Phi\cdot dA. \tag{10}$$

In the special limit $a_E/|\nabla\Phi| \to 0$ this equation reduces to the Gauss's law. But in general cases, it is not necessarily the Gauss's law, and we can write down its differential form

$$\nabla\cdot\left[\left(\frac{1}{2} + \frac{1}{e^{a_E/|\nabla\Phi|}-1}\right)\frac{a_E}{|\nabla\Phi|}\nabla\Phi\right] = 4\pi G\rho, \tag{11}$$

which is much more complicated than Poisson's equation.

IV. GEOMETRIZATION OF GRAVITY

A. Relativistic gravitational field equations

In the relativistic situation with a static background, we have a time-like Killing vector ξ^μ normalized as

$$\xi_\mu \xi^\mu = -e^{2\phi} \tag{12}$$

with the Newtonian potential ϕ. In this situation, the acceleration is a four vector $a^\mu = -\nabla^\mu \phi$, and equation (9) should be replaced with [2]

$$k_B T = \frac{\hbar}{2\pi c} e^{\phi} \mathcal{N}^\mu \nabla_\mu \phi. \tag{13}$$

Here $\mathcal{N}^\mu$ is the outward normal vector of the holographic screen satisfying $\mathcal{N}^\mu \xi_\mu = 0$.

Inserting equation (13) into the integral form of equipartition relation (8), we obtain

$$\begin{aligned} Mc^2 &= \frac{c^3}{2G\hbar} \int_{\mathcal{S}} \left(\frac{1}{2}\hbar\omega + \frac{\hbar\omega}{e^{\hbar\omega/k_B T} - 1} \right) \frac{\mathcal{N}^\mu \nabla_\mu \phi}{\mathcal{N}^\lambda \nabla_\lambda \phi} |dA| \\ &= \frac{a_E c^2}{4\pi G} \int_{\mathcal{S}} \left[\frac{1}{2} + \frac{1}{e^{a_E e^{-\phi}/(\mathcal{N}^\kappa \nabla_\kappa \phi)} - 1} \right] \frac{\nabla\phi}{\mathcal{N}^\lambda \nabla_\lambda \phi} \cdot dA, \end{aligned} \tag{14}$$

that is

$$M = \frac{1}{4\pi G} \int_{\mathcal{S}} \left[\frac{1}{2} + \frac{1}{e^{a_E e^{-\phi}/(\mathcal{N}^\kappa \nabla_\kappa \phi)} - 1} \right] \frac{a_E e^{-\phi}}{\mathcal{N}^\lambda \nabla_\lambda \phi} e^{\phi} \nabla\phi \cdot dA. \tag{15}$$

In the above, we have used the equality

$$dA^\mu = |dA| \mathcal{N}^\mu. \tag{16}$$

It is trivial to see $|dA| = dA_\mu \mathcal{N}^\mu$ as a result of the outward normalization condition $\mathcal{N}_\mu \mathcal{N}^\mu = 1$.

As demonstrated in appendix A, equation (15) can be rewritten in the form

$$M = \frac{1}{4\pi G} \int_{\mathcal{V}} (f \mathcal{R}_{\mu\nu} - \nabla_\mu \nabla_\nu f) n^\mu \xi^\nu dV \tag{17}$$

by virtue of the Stokes theorem, in which $\mathcal{V}$ is the 3-dimensional volume bounded by the holographic screen $\mathcal{S}$. In this expression, n^μ is a future-directed vector normal to $\mathcal{V}$, $\mathcal{R}_{\mu\nu}$ denotes the Ricci tensor, and

$$f = \left[\frac{1}{2} + \frac{1}{e^{a_E e^{-\phi}/(\mathcal{N}^\kappa \nabla_\kappa \phi)} - 1} \right] \frac{a_E e^{-\phi}}{\mathcal{N}^\lambda \nabla_\lambda \phi}. \tag{18}$$

On the other hand, the mass can be expressed as as a volume integral of the stress-energy tensor,

$$M = 2 \int_{\mathcal{V}} \left(\mathcal{T}_{\mu\nu} - \frac{1}{2} g_{\mu\nu} \mathcal{T} \right) n^\mu \xi^\nu dV. \tag{19}$$

Analogous to Jacobson's scheme [1], this directly leads to the integral relation

$$2 \int_{\mathcal{V}} \left(\mathcal{T}_{\mu\nu} - \frac{1}{2} g_{\mu\nu} \mathcal{T} \right) n^\mu \xi^\nu dV = \frac{1}{4\pi G} \int_{\mathcal{V}} (f \mathcal{R}_{\mu\nu} - \nabla_\mu \nabla_\nu f) n^\mu \xi^\nu dV \tag{20}$$

or equivalently

$$f\mathcal{R}_{\mu\nu} - \nabla_\mu \nabla_\nu f = 8\pi G \left(\mathcal{T}_{\mu\nu} - \frac{1}{2} g_{\mu\nu} \mathcal{T} \right). \tag{21}$$

In Verlinde's model, $f = 1$, and this equation reduces to the Einstein equations. In our model,

$$f = \left[\frac{1}{2} + \frac{1}{e^{a_E e^{-\phi}/(\mathcal{N}^\kappa \nabla_\kappa \phi)} - 1} \right] \frac{a_E e^{-\phi}}{\mathcal{N}^\lambda \nabla_\lambda \phi}, \tag{22}$$

which makes (21) very lengthy.

As emphasized in Appendix A, the form of gravitational field equations is not unique. Although (21) is advocated in this paper, we still keep an open mind to other choices. The non-uniqueness of gravitational equations was also noticed in [13] for Verlinde's primary model. Compared with (A18), gravitational equations of the form (21) have two merits. First, the left hand side is automatically symmetric with respect to indices μ and ν. Second, the Killing vector does not appears explicitly. Surely it is necessary to check the consistency of our choice. In the next subsection, we will check if (21) is consistent with the Newtonian limit obtained in previous sections.

B. Static weak field limit

In field equations (21), there are second-order derivatives of f. On the other hand, its derivative is of first order in the generalized Poisson equation (11). Then one would be worrying if our relativistic gravitational equations can reproduce the expected Newtonian limit correctly. This confusion can be resolved by noticing that the Newtonian limit corresponds to the 00-component of relativistic gravitational equations. For this component, the second term of the left hand side of (21) reads

$$-\nabla_0 \nabla_0 f = -\partial_0 \partial_0 f + \Gamma^\mu_{00} \partial_\mu f. \tag{23}$$

For the static case, $\partial_0 \partial_0 f$ vanishes and thus only the first-order derivative contributes.

Expanding weak gravitational field $g_{\mu\nu}$ near the Minkowski metric $\eta_{\mu\nu}$ as

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \quad |h_{\mu\nu}| \ll 1, \tag{24}$$

in the static case $\partial_0 g_{\mu\nu} = 0$ we have [14]

$$\Gamma^\mu_{00} = -\frac{1}{2} \eta^{\mu\lambda} \partial_\lambda h_{00}, \quad \mathcal{R}_{00} = -\frac{1}{2} \nabla^\lambda \nabla_\lambda h_{00} \tag{25}$$

where $h_{00} \simeq -2\phi$. Therefore the 00-component of (21) on the left hand side becomes

$$\begin{aligned} f\mathcal{R}_{00} - \nabla_0 \nabla_0 f &\simeq f \eta^{\mu\lambda} \partial_\mu \partial_\lambda \phi + \eta^{\mu\lambda} (\partial_\lambda \phi) \partial_\mu f \\ &= \eta^{\mu\lambda} \partial_\mu (f \partial_\lambda \phi) \end{aligned} \tag{26}$$

recovering equation (11) readily. This is good news: the gravitational equations we have chosen work consistently in the Newtonian limit.

One may also check that (A18) recovers the same Poisson's equation in this limit, though it is not preferred in this paper.

V. DISCUSSION

After Verlinde's proposal of entropic gravity, a lot of investigations have been done.[1] The equipartition rule for gravitational systems [4], proposed earlier than the idea of entropic gravity, takes an important part in deriving gravitational equations. From the lessons of thermodynamics and solid state physics, when quantum effects appear, this rule changes, with its exact form sensitive to the microscopic picture. Borrowing the idea of Einstein's solid model, in this paper we studied entropic gravity with the simplest model of quantum harmonic oscillators by modifying the equipartition rule.

Many of our results can be applied to a larger class of models. For instance, when the equipartition relation is deformed as $E = f(T)Nk_BT/2$, all results in Appendix A continue to hold. In this sense, although our motivation is different, some of our results in this paper overlap with the so-called Debye entropic gravity [15–19]. In the solid state physics, Debye model works better than Einstein model, but this is not guaranteed for gravity. Equations of the form (A18) were recently obtained in [19], but in our paper, after some algebras, we got a more symmetric, cleaner form, namely (21).

The quantum oscillator model of entropic gravity presented in this paper is a toy model. The Debye entropic gravity can be also taken as a toy model. Before clarifying the microscopic picture of bits/degrees of freedom in entropic force, we may keep an open mind to possible modifications of the equipartition relation. It is not meaningless to generally study how far could such modifications go and how to constrain them theoretically or phenomenologically. We will return to this problem in the future.

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Appendix A: Calculation of Mass

In this appendix, we will derive equation (17) from (15). Before our calculation, it is convenient to uplift expression (15) to a generic form

$$M = \frac{1}{4\pi G}\int_{\mathcal{S}} f e^{\phi}\nabla\phi \cdot dA, \tag{A1}$$

in which f is a model-dependent function. In Verlinde's model, $f = 1$. In our model, f is given by expression (22). In this appendix, we will leave f as a generic function before specifying it.

It is remarkable that the magnitude of dA is

$$|dA| = dx^{\alpha}dx^{\beta}\epsilon_{\kappa\lambda\alpha\beta}e^{-\phi}\xi^{\kappa}\mathcal{N}^{\lambda}, \tag{A2}$$

[1] See [20–31] as a short list and references therein.

but as a four-vector,

$$dA_\mu = |dA|\mathcal{N}_\mu = dx^\alpha dx^\beta \epsilon_{\kappa\lambda\alpha\beta} e^{-\phi}\xi^\kappa \mathcal{N}^\lambda \mathcal{N}_\mu. \tag{A3}$$

For ξ^μ, we have the Killing equation

$$\nabla^\mu \xi^\nu + \nabla^\nu \xi^\mu = 0 \tag{A4}$$

and the formula

$$\nabla_\mu \nabla^\mu \xi^\nu = -\mathcal{R}^\nu{}_\mu \xi^\mu, \tag{A5}$$

where $\mathcal{R}^\nu{}_\mu$ is the Ricci tensor. From normalization condition (12), we find

$$e^{2\phi}\nabla^\mu \phi = \frac{1}{2}\nabla^\mu(-\xi_\nu \xi^\nu) = -\xi_\nu \nabla^\mu \xi^\nu. \tag{A6}$$

Utilizing equations (A3), (A4) and (A6), it is easy to show that

$$\begin{aligned} M &= \frac{1}{4\pi G}\int_{\mathcal{S}} dx^\alpha dx^\beta \epsilon_{\kappa\lambda\alpha\beta} e^{-\phi}\xi^\kappa \mathcal{N}^\lambda f \mathcal{N}_\mu e^\phi \nabla^\mu \phi \\ &= \frac{1}{4\pi G}\int_{\mathcal{S}} dx^\alpha dx^\beta \epsilon_{\kappa\lambda\alpha\beta} e^{-2\phi}\xi^\kappa \mathcal{N}^\lambda f \mathcal{N}_\mu (-\xi_\nu \nabla^\mu \xi^\nu) \\ &= \frac{1}{4\pi G}\int_{\mathcal{S}} dx^\alpha dx^\beta \epsilon_{\kappa\lambda\alpha\beta} e^{-2\phi}\xi^\kappa \mathcal{N}^\lambda f \mathcal{N}_\mu \xi_\nu \nabla^\nu \xi^\mu. \end{aligned} \tag{A7}$$

To proceed, we note that the holographic screen $\mathcal{S}$ as a 2-dimensional surface. Restricted to this surface, we can define a 2-dimensional tensor $\omega^{\alpha\beta} = \xi_\kappa N_\lambda \epsilon^{\kappa\lambda\alpha\beta}$. Making use of the equality

$$\epsilon^{\kappa\lambda\alpha\beta}\epsilon_{\mu\nu\alpha\beta} = -4\delta^\kappa{}_{[\mu}\delta^\lambda{}_{\nu]} = \frac{-4}{2!}(\delta^\kappa{}_\mu \delta^\lambda{}_\nu - \delta^\kappa{}_\nu \delta^\lambda{}_\mu), \tag{A8}$$

one may directly prove

$$\omega^{\alpha\beta}\epsilon_{\kappa\lambda\alpha\beta} e^{-2\phi}\xi^\kappa \mathcal{N}^\lambda f \mathcal{N}_\mu \xi_\nu \nabla^\nu \xi^\mu = -\frac{1}{2}\omega^{\alpha\beta}\epsilon_{\mu\nu\alpha\beta} f \nabla^\mu \xi^\nu. \tag{A9}$$

In addition, restricted to the holographic screen $\mathcal{S}$, all 2-dimensional anti-symmetric covariant tensor should be proportional to $\omega_{\alpha\beta}$ and thus proportional to each other in the same basis. As a result, we conclude that

$$\epsilon_{\kappa\lambda\alpha\beta} e^{-2\phi}\xi^\kappa \mathcal{N}^\lambda f \mathcal{N}_\mu \xi_\nu \nabla^\nu \xi^\mu = -\frac{1}{2}\epsilon_{\mu\nu\alpha\beta} f \nabla^\mu \xi^\nu \tag{A10}$$

and subsequently

$$\begin{aligned} M &= -\frac{1}{8\pi G}\int_{\mathcal{S}} dx^\alpha dx^\beta \epsilon_{\mu\nu\alpha\beta} f \nabla^\mu \xi^\nu \\ &= -\frac{1}{8\pi G}\int_{\mathcal{S}} dx^\alpha dx^\beta \epsilon_{\alpha\beta\mu\nu} f \nabla^\mu \xi^\nu \\ &= -\frac{3}{8\pi G}\int_{\mathcal{V}} dx^\alpha dx^\beta dx^\gamma \nabla_{[\gamma}(\epsilon_{\alpha\beta]\mu\nu} f \nabla^\mu \xi^\nu) \\ &= \frac{1}{16\pi G}\int_{\mathcal{V}} dx^\alpha dx^\beta dx^\gamma \epsilon_{\rho\gamma\alpha\beta}\epsilon^{\rho\sigma\kappa\lambda}\nabla_\sigma(\epsilon_{\kappa\lambda\mu\nu} f \nabla^\mu \xi^\nu). \end{aligned} \tag{A11}$$

Here $\mathcal{V}$ is the volume enclosed by the holographic screen $\mathcal{S}$. In the third line, the Stokes theorem was applied. In the last step, we have made use of

$$\epsilon_{\rho\gamma\alpha\beta}\epsilon^{\rho\sigma\kappa\lambda} = -6\delta^{\sigma}{}_{[\gamma}\delta^{\kappa}{}_{\alpha}\delta^{\lambda}{}_{\beta]}. \tag{A12}$$

With the help of equations (A4), (A5) and (A8), we can demonstrate that

$$\begin{aligned}\epsilon^{\rho\sigma\kappa\lambda}\nabla_{\sigma}(\epsilon_{\kappa\lambda\mu\nu}f\nabla^{\mu}\xi^{\nu}) &= \epsilon^{\rho\sigma\kappa\lambda}\epsilon_{\kappa\lambda\mu\nu}\nabla_{\sigma}(f\nabla^{\mu}\xi^{\nu}) \\ &= -4\nabla_{\sigma}(f\nabla^{[\rho}\xi^{\sigma]}) \\ &= 4(\nabla_{\sigma}f)\nabla^{\sigma}\xi^{\rho} + 4f\nabla_{\sigma}\nabla^{\sigma}\xi^{\rho} \\ &= 4(\nabla_{\sigma}f)\nabla^{\sigma}\xi^{\rho} - 4f\mathcal{R}^{\rho}{}_{\sigma}\xi^{\sigma}.\end{aligned} \tag{A13}$$

Substituted into (A11), it yields

$$M = \frac{1}{4\pi G}\int_{\mathcal{V}} dx^{\alpha}dx^{\beta}dx^{\gamma}\epsilon_{\rho\gamma\alpha\beta}[(\nabla_{\sigma}f)\nabla^{\sigma}\xi^{\rho} - f\mathcal{R}^{\rho}{}_{\sigma}\xi^{\sigma}]. \tag{A14}$$

Let us introduce a future-directed vector n^{μ} normal to the 3-dimensional hyper-surface $\mathcal{V}$. Restricted to this hyper-surface, all 3-dimensional totally-anti-symmetric covariant tensor are proportional to $\omega_{\alpha\beta\gamma} = n^{\kappa}\epsilon_{\kappa\gamma\alpha\beta}$. The vector n^{μ}, normalized as $n_{\mu}n^{\mu} = -1$, should not be confused with $\mathcal{N}^{\mu}$. According to (A8), we have

$$\omega^{\alpha\beta\gamma}\epsilon_{\rho\gamma\alpha\beta}[(\nabla_{\sigma}f)\nabla^{\sigma}\xi^{\rho} - f\mathcal{R}^{\rho}{}_{\sigma}\xi^{\sigma}] = \omega^{\alpha\beta\gamma}\epsilon_{\kappa\gamma\alpha\beta}n^{\kappa}[f\mathcal{R}^{\rho}{}_{\sigma}\xi^{\sigma}n_{\rho} - (\nabla_{\sigma}f)(\nabla^{\sigma}\xi^{\rho})n_{\rho}] \tag{A15}$$

and consequently

$$\begin{aligned}M &= \frac{1}{4\pi G}\int_{\mathcal{V}} dx^{\alpha}dx^{\beta}dx^{\gamma}\epsilon_{\kappa\gamma\alpha\beta}n^{\kappa}[f\mathcal{R}^{\rho}{}_{\sigma}\xi^{\sigma}n_{\rho} - (\nabla_{\sigma}f)(\nabla^{\sigma}\xi^{\rho})n_{\rho}] \\ &= \frac{1}{4\pi G}\int_{\mathcal{V}}[f\mathcal{R}^{\rho}{}_{\sigma}\xi^{\sigma}n_{\rho} + (\nabla_{\sigma}f)(\nabla^{\rho}\xi^{\sigma})n_{\rho}]dV\end{aligned} \tag{A16}$$

with the volume element $dV = \epsilon_{\kappa\gamma\alpha\beta}n^{\kappa}dx^{\alpha}dx^{\beta}dx^{\gamma}$. Here the Killing equation (A4) was used.

From expression (A16), there is ambiguity in building the gravitational field equations. For example, one may straightforwardly rewrite it as

$$\begin{aligned}M &= \frac{1}{4\pi G}\int_{\mathcal{V}}[f\mathcal{R}_{\mu\nu}n^{\mu}\xi^{\nu} - (\nabla_{\sigma}f)(\nabla_{\mu}\xi^{\sigma})n^{\mu}\xi^{\nu}\xi_{\nu}e^{-2\phi}]dV \\ &= \frac{1}{4\pi G}\int_{\mathcal{V}}[f\mathcal{R}_{\mu\nu} - (\nabla_{\sigma}f)(\nabla_{\mu}\xi^{\sigma})\xi_{\nu}e^{-2\phi}]n^{\mu}\xi^{\nu}dV\end{aligned} \tag{A17}$$

with the help of equation (12). For entropic gravity inspired from the Debye solid model, expression (A17) was also obtained in [19] when our work is under progress. It is different from equation (17) in section IV. Instead of (21), it leads to the following form of gravitational field equations:

$$f\mathcal{R}_{\mu\nu} - (\nabla_{\sigma}f)(\nabla_{\mu}\xi^{\sigma})\xi_{\nu}e^{-2\phi} = 8\pi G\left(\mathcal{T}_{\mu\nu} - \frac{1}{2}g_{\mu\nu}\mathcal{T}\right). \tag{A18}$$

However, we do not advocate this form of equations, because the Killing vector ξ^{σ} appears in them explicitly.

Alternatively, we prefer to recast the second term of (A16) into

$$(\nabla_\sigma f)(\nabla^\rho \xi^\sigma) n_\rho = n_\rho \nabla^\rho (\xi^\sigma \nabla_\sigma f) - n_\rho \xi^\sigma \nabla^\rho \nabla_\sigma f. \tag{A19}$$

In what follows, we will prove $\xi^\sigma \nabla_\sigma f = 0$ if f is a function of the Unruh temperature (13). First, according to equations (A4) and (A6), we have

$$\xi^\mu \nabla_\mu \phi = -e^{-2\phi} \xi^\mu \xi^\nu \nabla_\mu \xi_\nu = 0. \tag{A20}$$

Second, it is easy to see

$$\nabla_\sigma \nabla_\mu e^{2\phi} = \nabla_\mu \nabla_\sigma e^{2\phi}, \quad \mathcal{N}^\mu \nabla^\nu \xi_\mu = \xi_\mu \nabla^\nu \mathcal{N}^\mu. \tag{A21}$$

Taking equations (A4), (A6), (A20) and (A21) into consideration, straight forward calculations yield

$$\begin{aligned}
\xi^\sigma \nabla_\sigma (e^\phi \mathcal{N}^\mu \nabla_\mu \phi) &= e^\phi \xi^\sigma (\nabla_\mu \phi) \nabla_\sigma \mathcal{N}^\mu + \frac{1}{2} e^{-\phi} \xi^\sigma \mathcal{N}^\mu \nabla_\sigma \nabla_\mu e^{2\phi} \\
&= e^\phi \xi^\sigma (\nabla_\mu \phi) \nabla_\sigma \mathcal{N}^\mu - e^{-\phi} \xi^\sigma \mathcal{N}^\mu \nabla_\mu (\xi^\nu \nabla_\sigma \xi_\nu) \\
&= e^\phi \xi^\sigma (\nabla_\mu \phi) \nabla_\sigma \mathcal{N}^\mu - e^{-\phi} \xi^\sigma \mathcal{N}^\mu (\nabla_\mu \xi^\nu) \nabla_\sigma \xi_\nu - e^{-\phi} \xi^\sigma \mathcal{N}^\mu \xi^\nu \nabla_\mu \nabla_\sigma \xi_\nu \\
&= e^\phi \xi^\sigma (\nabla_\mu \phi) \nabla_\sigma \mathcal{N}^\mu - e^{-\phi} \xi^\sigma \mathcal{N}^\mu (\nabla^\nu \xi_\mu) \nabla_\nu \xi_\sigma \\
&= e^\phi \xi_\mu (\nabla_\nu \phi) \nabla^\mu \mathcal{N}^\nu + e^\phi \mathcal{N}^\mu (\nabla^\nu \xi_\mu) \nabla_\nu \phi \\
&= e^\phi (\nabla_\nu \phi) \left(\xi_\mu \nabla^\mu \mathcal{N}^\nu + \mathcal{N}^\mu \nabla^\nu \xi_\mu \right).
\end{aligned} \tag{A22}$$

Remember that the holographic screen $\mathcal{S}$ corresponds to the equipotential surface [2], so we can expand the gradient of the Newtonian potential in directions normal to $\mathcal{S}$ as

$$\nabla_\nu \phi = -e^{-2\phi} \xi_\nu \xi_\lambda \nabla^\lambda \phi + \mathcal{N}_\nu \mathcal{N}_\lambda \nabla^\lambda \phi. \tag{A23}$$

Comparing it with (A20), we see the gradient of Newtonian potential aligns along the vector $\mathcal{N}_\nu$, namely $\nabla_\nu \phi \propto \mathcal{N}_\nu$. We substitute it into (A22), and notice

$$\mathcal{N}_\nu \nabla^\mu \mathcal{N}^\nu = 0, \quad \mathcal{N}_\nu \mathcal{N}_\mu \nabla^\nu \xi^\mu = 0, \tag{A24}$$

then it is clear that

$$\xi^\sigma \nabla_\sigma f \propto \xi^\sigma \nabla_\sigma (e^\phi \mathcal{N}^\mu \nabla_\mu \phi) \propto \mathcal{N}_\nu \left(\xi_\mu \nabla^\mu \mathcal{N}^\nu + \mathcal{N}^\mu \nabla^\nu \xi_\mu \right) = 0. \tag{A25}$$

With $\xi^\sigma \nabla_\sigma f = 0$ demonstrated, we can see the first term of (A19) vanishes. As a result, the expression for mass (A16) now becomes

$$\begin{aligned}
M &= \frac{1}{4\pi G} \int_{\mathcal{V}} (f \mathcal{R}^\rho{}_\sigma \xi^\sigma n_\rho - n_\rho \xi^\sigma \nabla^\rho \nabla_\sigma f) dV \\
&= \frac{1}{4\pi G} \int_{\mathcal{V}} (f \mathcal{R}_{\mu\nu} - \nabla_\mu \nabla_\nu f) n^\mu \xi^\nu dV
\end{aligned} \tag{A26}$$

This gives equation (17) in section IV.

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